Obstructions: Comparing Circles to Squares

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In the thread called "Re: [osw] Building alignment in" someone wrote

>>> Just do not underestimate that you cannot compare a central round obstruction and a side square one. The Strehl ratio is different.

I responded with a half-baked comparison and I wish now to provide a full explanation.

Strehl Ratio

The term is tossed out in discussion with an assumption that both the speaker and listener know and agree on what it means. The definition is:

1. "The Strehl ratio is frequently defined as the ratio of the peak aberrated image intensity from a point source compared to the maximum attainable intensity using an ideal optical system limited only by diffraction over the system's aperture." *Sacek, Vladimir (July 14, 2006), "6.5. Strehl ratio", Notes on amateur telescope optics, retrieved March 2, 2011*

While this is very useful if we know the wavefront error of a telescope mirror, there is a subtlety that needs explanation for the example being discussed, which is the effect of obstruction. The first part of the definition speaks to an aberrated image. In my calculations of the effects of obstructions the telescope model has zero aberrations on axis, it is a perfect paraboloid. The last part of the definition "limited only by diffraction over the system's aperture" is therefore very important. What it means is that the Strehl ratio is calculated relative to the apertures and obstructions present in the system. If an obstruction is introduced, the Strehl ratio does not change because the optical design software (OSLO-EDU) does not consider the unobstructed aperture. One instead has to find a manual process.

Modulation Transfer Function (MTF)

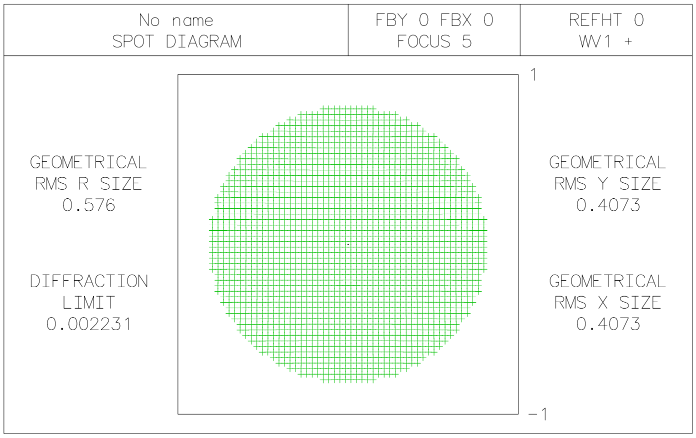
Instead of using Strehl Ratio, I chose to show MTF charts because they can be easily compared for unobstructed and obstructed apertures with a perfect aberration-free scope. MTF is a ratio similar to Strehl with the addition of calculations as a function of spatial frequency in the image as well as a plot of the diffraction limit. Because the telescope is diffraction limited no matter what obstructions are applied MTF is only useful if we *compare the diffraction limits themselves.*

Examples with Explanations

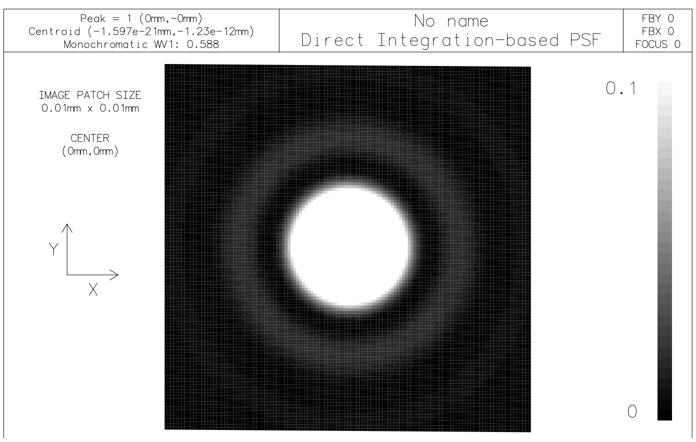
I will use my model of a 16" F/3 paraboloid for these examples. The first set will show unobstructed aperture, then I will compare a round secondary and a rectangular bar (emulating a focuser intrusion.)

1. Unobstructed

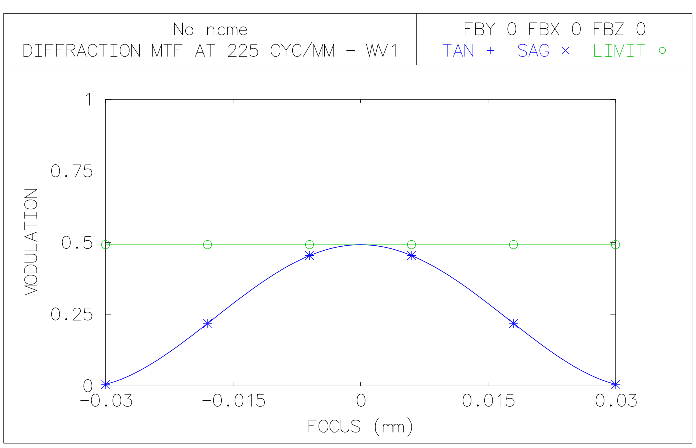
A spot diagram taken 5 inches outside of focus shows that the mirror aperture is unobstructed (and perfectly round):



The Airy Disk is clearly seen with a ring:



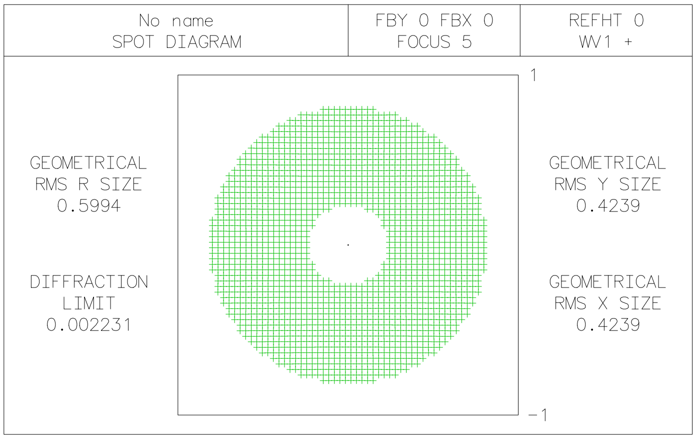
And the through-focus MTF is plotted at a high spatial frequency, 225 cycles/mm, which corresponds roughly to the Rayleigh criterion.



It is most important to note that the diffraction limit is 0.493 (green line).

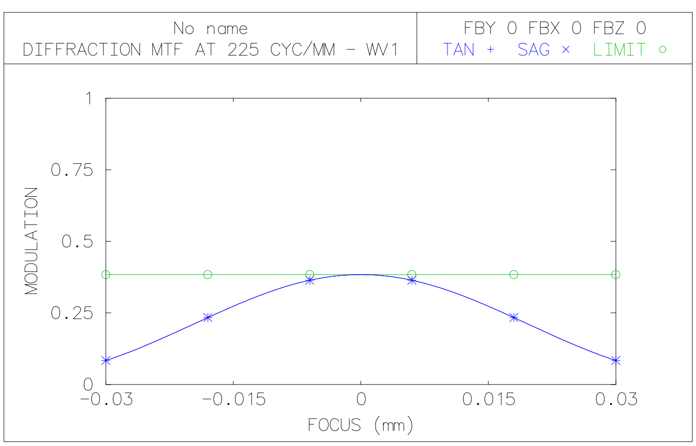
2. 4.7" Obstruction

The secondary mirror is now added. A central hole shows up clearly in the spot diagram.



The first ring around the Airy disk is much brighter, as expected.

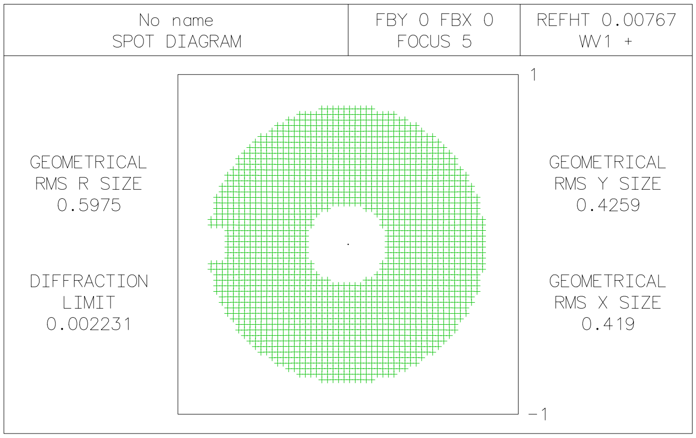
And the diffraction-limited MTF has dropped quite a bit! = 0.384



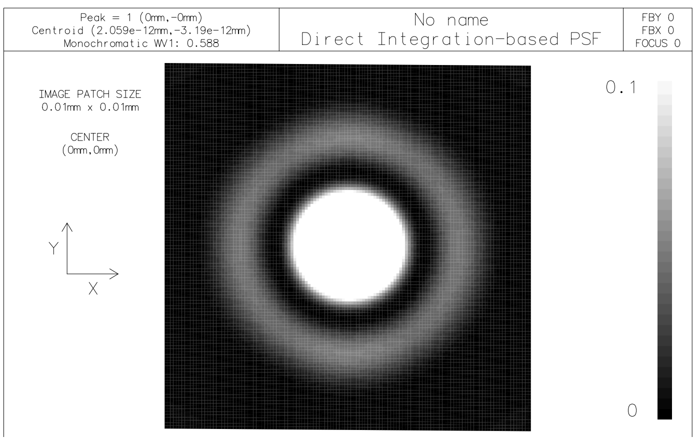
This is of course the reason why we want to keep our secondaries as small as practicable.

3. Focuser Intrusion

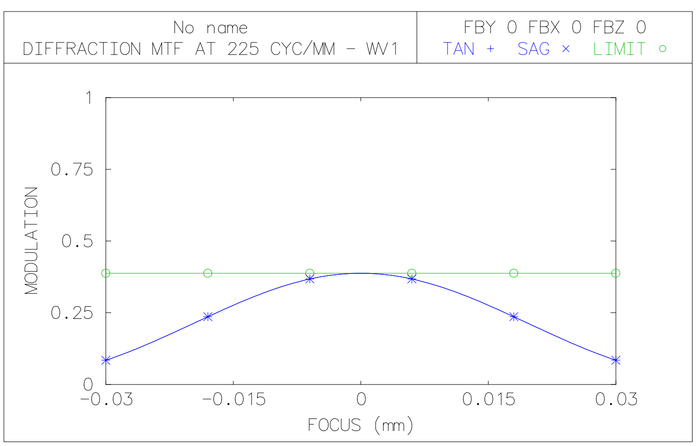
And now for the act we've been waiting for, a focuser that intrudes into the light path. I assumed a 2" diameter cylinder poking into the light path 1":



Barely noticeable are two bright spots at top and bottom of the first diffraction ring:



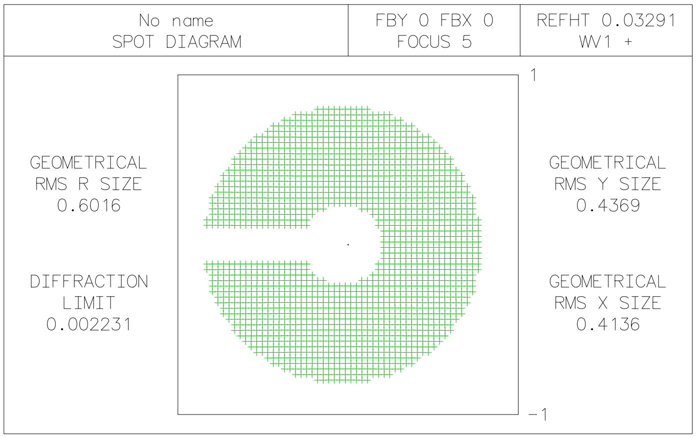
And the MTF is unchanged. (Honestly, the limit is now 0.388, slightly *higher* as calculated)



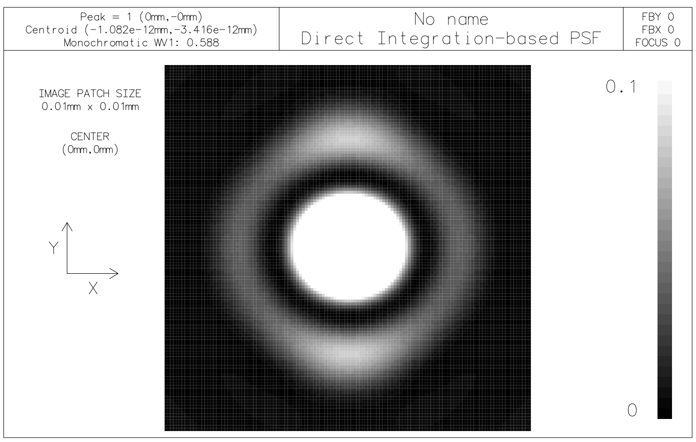
Mel's argument that a small intrusion makes little impact holds true. But if it's worth doing then it's worth overdoing, so…

4. Big Giant Secondary Stalk

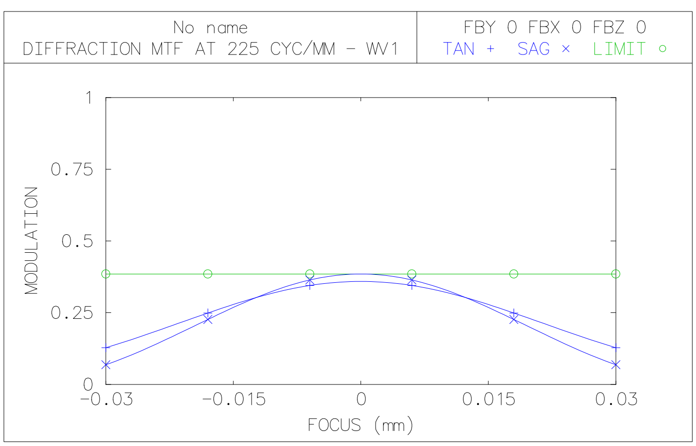
In this example one could hide the focuser intrusion entirely behind the stalk!



Big bright spots appear in the first ring. Large diffraction spikes radiate from these spots but are hard to show in this tool.



And the MTF has dropped to 0.360



Analysis of Results

A quick summary of the diffraction limits for four examples:

Example	Limit at 225 c/mm
Unobstructed	0.493
Obstructed, 4.7”	0.384
Obstructed + Intrusion 1”	0.388
Obstructed + Big Giant Stalk	0.360

So, what does the obstruction do to the Strehl ratio? By the textbook definition absolutely nothing, but when comparing different diffraction limits…not much! Let’s do the math:

Example	Limit at 225 c/mm	Ratio of Limits
Unobstructed	0.493	1.000
Obstructed, 4.7”	0.384	0.779
Obstructed + Intrusion 1”	0.388	0.787
Obstructed + Big Giant Stalk	0.360	0.730

Given that the obstruction is required for this Newtonian design, we can eliminate the unobstructed example and get a new set of ratios to compare the addition of the focuser intrusion:

Example	Limit at 225 c/mm	Ratio of Limits
Obstructed, 4.7”	0.384	1.000
Obstructed + Intrusion 1”	0.388	1.010
Obstructed + Big Giant Stalk	0.360	0.923

The Strehl ratio is not affected with a marginally intruding secondary.

MTF in these examples is taken through a vertical slice of the Airy disk. Note that in the last example a horizontal slice would encounter the same irradiance profile as the obstruction without intrusion (which is rotationally symmetric.) The MTF in the horizontal cross section for a large stalk is unaffected, 0.384, and therefore the Strehl in this direction is 1.000. What this means (and should be obvious by looking at the Airy disk) is that there are two Rayleigh criteria for a scope with asymmetric obstructions! Since Strehl ratio doesn’t ever mention this, it’s not a good metric. I think it’s better to compare the Rayleigh criteria in two axes, but that’s going to be another long chapter to be written some other time, if ever.

Strehl ratio is meaningful when looking at marginally *aberrated* optics, including off-axis situations and mirror manufacturing tolerances. It isn’t a meaningful metric for comparing apertures and obstructions, as shown here, and further supported by the fact that perfect optics in a 2” scope and perfect optics in a 20” scope both have a Strehl ratio of 1.0.